Proof of Horner’s Rule Correctness

- **Loop Invariant**

  \[ y = \sum_{k=0}^{n-(i+1)} a_{k+i+1}x^k \]

- **Initialization** Before the loop \( y = 0 \). At initialization, \( i = n \).

  \[ y = \sum_{k=0}^{n-(n+1)} a_{k+i+1}x^k = \sum_{k=0}^{-1} a_{k+i+1}x^k = 0 \]
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- **Loop Invariant**

\[ y = \sum_{k=0}^{n-(i+1)} a_{k+i+1} x^k \]

- **Maintenance** At iteration \( j \) of the loop, \( i' = n - j + 1 \) and 
\[ y' = \sum_{k=0}^{n-(i'+1)} a_{k+i'+1} x^k \]. At iteration \( j + 1 \), \( i = n - j \) and 
\[ y = a_i + xy' \]. Need to show 
\[ y = \sum_{k=0}^{n-i} a_{k+i} x^k \]
Maintenance

Need to show

\[ y = \sum_{k=0}^{n-i} a_{k+i}x^k \]

\[ y = a_i + x \left( \sum_{k=0}^{n-(i+1)} a_{k+i+1}x^k \right) \]  

(1)

\[ = a_i + x \left( a_{0+i+1}x^0 + \ldots + a_{(n-i)+i}x^{n-(i+1)} \right) \]  

(2)

\[ = a_i + (a_{0+i+1}x^1 + \ldots + a_{(n-i)+i}x^{n-i}) \]  

(3)

\[ = a_i + \sum_{k=1}^{n-i} a_{k+i}x^k \]  

(4)

\[ = a_{i+0}x^0 + \sum_{k=1}^{n-i} a_{k+i}x^k \]  

(5)

\[ = \sum_{k=0}^{n-i} a_{k+i}x^k \]  

(6)
Proof of Horner’s Rule Correctness

- **Loop Invariant**

  \[ y = \sum_{k=0}^{n-(i+1)} a_{k+i+1}x^k \]

- **Termination** At the end of the loop, \( i = -1 \). Therefore

  \[ y = \sum_{k=0}^{n-(i+1)} a_{k+i+1}x^k = \sum_{k=0}^{n} a_kx^k \]