Lecture 22: Evaluation

April 24, 2010

Last Time

- Spectral Clustering

Today

- Evaluation Measures
  - Accuracy
  - Significance Testing
  - F-Measure
  - Error Types
    - ROC Curves
    - Equal Error Rate
  - AIC/BIC

How do you know that you have a good classifier?

- Is a feature contributing to overall performance?
- Is classifier A better than classifier B?
- Internal Evaluation:
  - Measure the performance of the classifier.
- External Evaluation:
  - Measure the performance on a downstream task

Accuracy

- Easily the most common and intuitive measure of classification performance.

$$Accuracy = \frac{\#correct}{N}$$

Significance testing

- Say I have two classifiers.

- A = 50% accuracy
- B = 75% accuracy

- B is better, right?

Significance Testing

- Say I have another two classifiers

- A = 50% accuracy
- B = 50.5% accuracy

- Is B better?

Basic Evaluation

- Training data – used to identify model parameters
- Testing data – used for evaluation
- Optionally: Development / tuning data – used to identify model hyperparameters.

- Difficult to get significance or confidence values

Cross validation

- Identify *n* “folds” of the available data.
- Train on *n-1* folds
- Test on the remaining fold.

- In the extreme (*n=N*) this is known as “**leave-one-out**” cross validation

- *n*-fold cross validation (xval) gives *n* samples of the performance of the classifier.

Significance Testing

- Is the performance of two classifiers different with statistical significance?
- Means testing
  - If we have two samples of classifier performance (accuracy), we want to determine if they are drawn from the same distribution (no difference) or two different distributions.

T-test

- One Sample t-test

$$t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}},$$

Once you have a t-value, look up the significance level on a table, keyed on the t-value and degrees of freedom

- Independent t-test
  - Unequal variances and sample sizes

$$t = \frac{\overline{X}_1 - \overline{X}_2}{s_{\overline{X}_1 - \overline{X}_2}}$$

$$s_{\overline{X}_1 - \overline{X}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}.$$

Significance Testing

- Run Cross-validation to get n-samples of the classifier mean.
- Use this distribution to compare against either:
  - A known (published) level of performance
    - one sample t-test
  - Another distribution of performance
    - two sample t-test
- If at all possible, results should include information about the variance of classifier performance.

Significance Testing

- Caveat – including more samples of the classifier performance can artificially inflate the significance measure.
  - If x and s are constant (the sample represents the population mean and variance) then raising n will increase t.
  - If these samples are real, then this is fine. Often cross-validation fold assignment is not truly random. Thus subsequent xval runs only resample the same information.

$$t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}},$$

Confidence Bars

- Variance information can be included in plots of classifier performance to ease visualization.

$$\mu = 10 \qquad \sigma = 1 \qquad n = 10$$

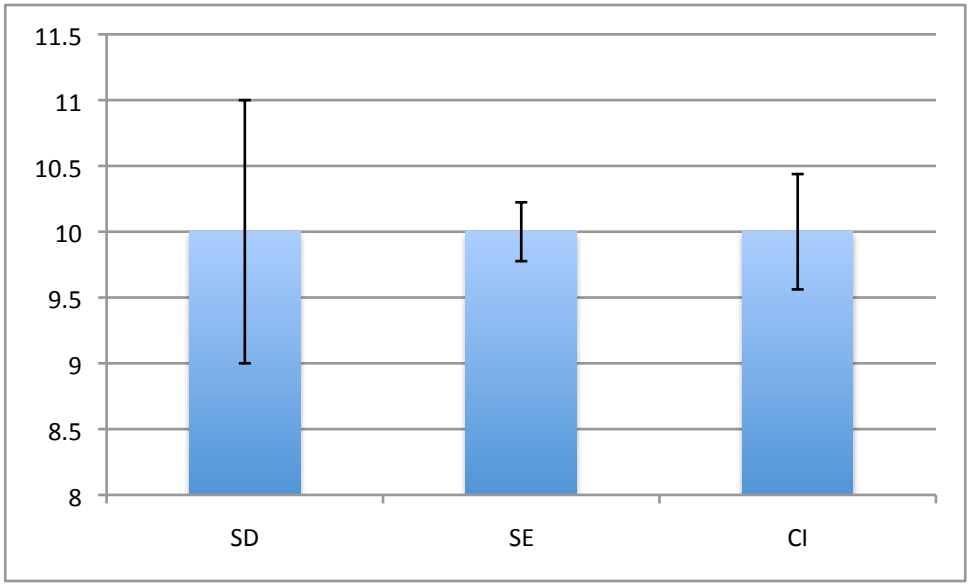
- Plot standard deviation, standard error or confidence interval?

$$SD = \sigma \qquad SE = \frac{\sigma}{\sqrt{n}}$$

$$CI_{95\%} = \mu \pm 1.96 * \frac{\sigma}{\sqrt{n}}$$

Confidence Bars

- Most important to be clear about what is plotted.
- 95% confidence interval has the clearest interpretation.



Baseline Classifiers

- Majority Class baseline
  - Every data point is classified as the class that is most frequently represented in the training data
- Random baseline
  - Randomly assign one of the classes to each data point.
    - with an even distribution
    - with the training class distribution

Problems with accuracy

- Contingency Table

		True Values	
		Positive	**Negative**
Hyp Values	**Positive**	True Positive	False Positive
	Negative	False Negative	True Negative

$$Accuracy = \frac{TP + TN}{TP + FP + TN + FN}$$

Problems with accuracy

- Information Retrieval Example
  - Find the 10 documents related to a query in a set of 110 documents

		True Values	
		Positive	Negative
Hyp Values	Positive	0	0
	Negative	10	100

$$Accuracy = 90\%$$

Problems with accuracy

- Precision: how many hypothesized events were true events
- Recall: how many of the true events were identified
- F-Measure: Harmonic mean of precision and recall

$$P = \frac{TP}{TP + FP}$$

$$R = \frac{TP}{TP + FN}$$

$$F = \frac{2PR}{P + R}$$

		True Values	
		Positive	Negative
Hyp Values	Positive	0	0
	Negative	10	100

F-Measure

- F-measure can be weighted to favor Precision or Recall
  - beta > 1 favors recall
  - beta < 1 favors precision

$$F_\beta = \frac{(1+\beta^2)PR}{(\beta^2 P) + R}$$

		True Values	
		Positive	Negative
Hyp Values	Positive	0	0
	Negative	10	100

$$P = 0$$

$$R = 0$$

$$F_1 = 0$$

F-Measure

		True Values	
		Positive	Negative
Hyp Values	Positive	1	0
	Negative	9	100

$$F_\beta = \frac{(1+\beta^2)PR}{(\beta^2 P) + R}$$

$$\begin{aligned} P &= 1 \\ R &= \frac{1}{10} \\ F_1 &= .18 \end{aligned}$$

F-Measure

		True Values	
		Positive	Negative
Hyp Values	Positive	10	50
	Negative	0	50

$$F_\beta = \frac{(1+\beta^2)PR}{(\beta^2 P) + R}$$

$$P = \frac{10}{60}$$

$$R = 1$$

$$F_1 = .29$$

F-Measure

		True Values	
		Positive	Negative
Hyp Values	Positive	9	1
	Negative	1	99

$$F_\beta = \frac{(1+\beta^2)PR}{(\beta^2 P) + R}$$

$$P = .9$$

$$R = .9$$

$$F_1 = .9$$

F-Measure

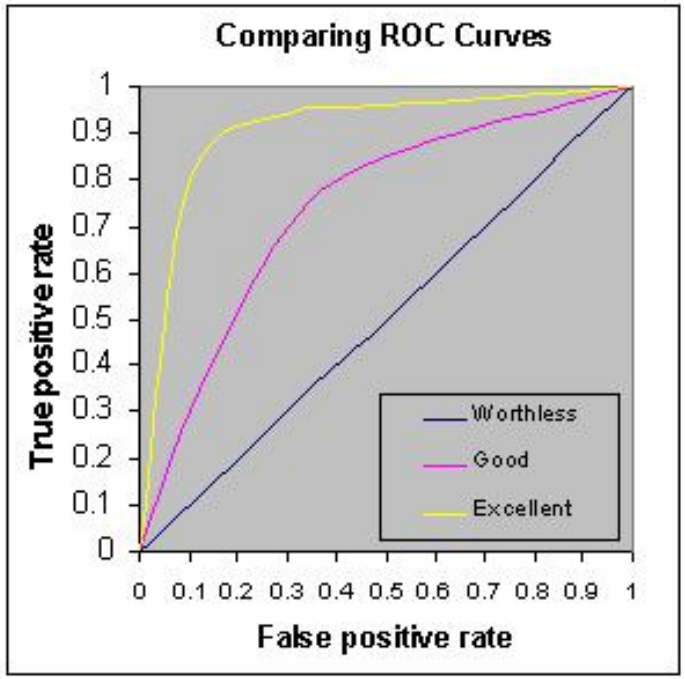
- Accuracy is weighted towards majority class performance.
- F-measure is useful for measuring the performance on minority classes.

Types of Errors

- False Positives
  - The system predicted **TRUE** but the value was **FALSE**
  - aka “False Alarms” or Type I error
- False Negatives
  - The system predicted **FALSE** but the value was **TRUE**
  - aka “Misses” or Type II error

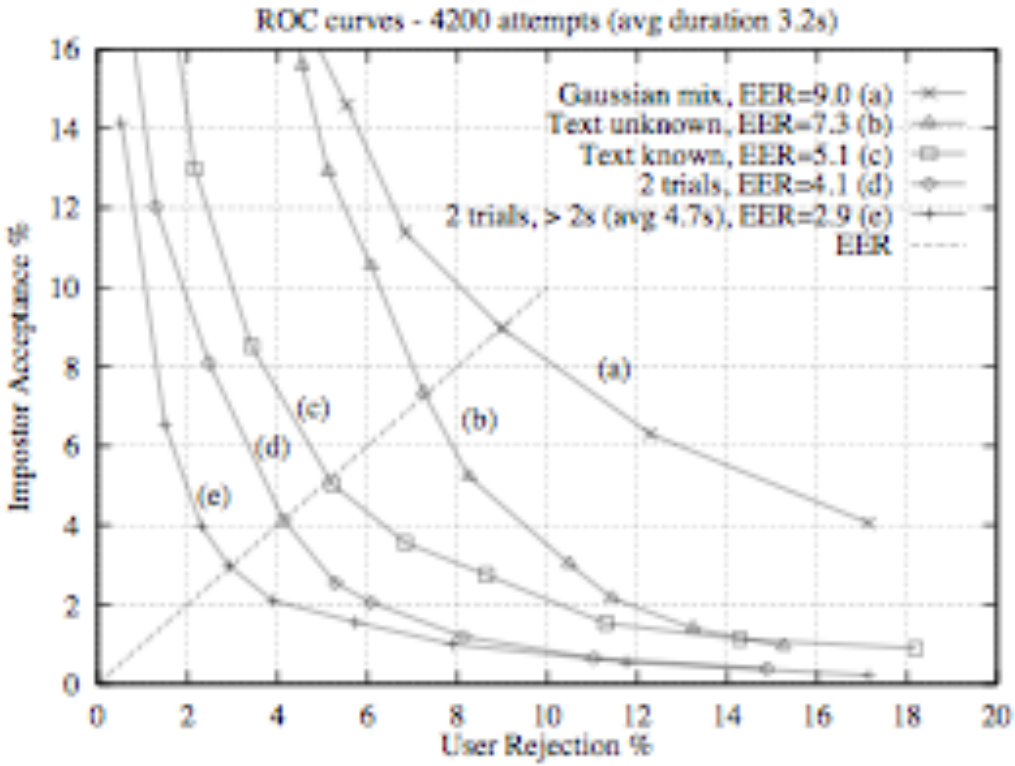
ROC curves

- It is common to plot classifier performance at a variety of settings or thresholds
- Receiver Operating Characteristic (ROC) curves plot true positives against false positives.
- The overall performance is calculated by the Area Under the Curve (AUC)



ROC Curves

- Equal Error Rate (EER) is commonly reported.
- EER represents the highest accuracy of the classifier
- Curves provide more detail about performance



Gauvain et al. 1995

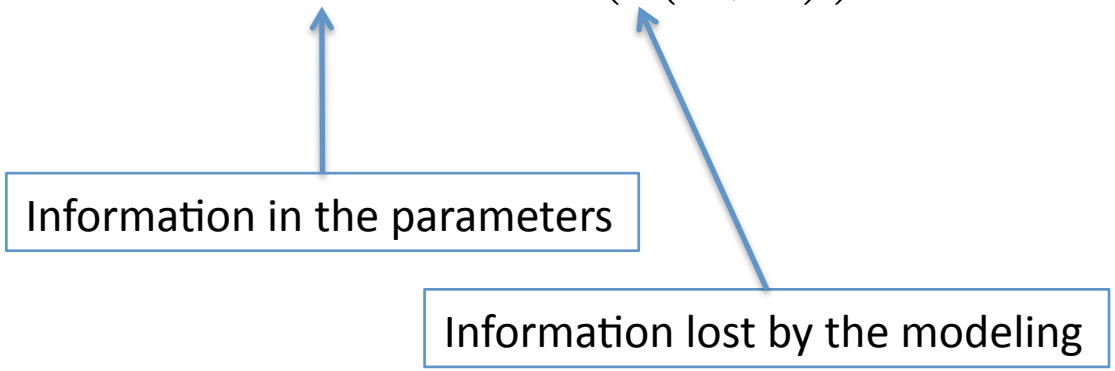
Goodness of Fit

- Another view of model performance.
- Measure the model likelihood of the unseen data. $l(x;\theta)$
- However, we've seen that model likelihood is likely to improve by adding parameters.
- Two information criteria measures include a cost term for the number of parameters in the model

Akaike Information Criterion

- Akaike Information Criterion (AIC) based on entropy
- The best model has the lowest AIC.
  - Greatest model likelihood
  - Fewest free parameters

$$AIC = 2k - 2\ln(l(x;\theta))$$



Bayesian Information Criterion

- Another penalization term based on Bayesian arguments
  - Select the model that is *a posteriori* most probably with a constant penalty term for wrong models

$$BIC = k\ln(n) - 2\ln(l(x;\theta))$$

  - If errors are normally distributed.

$$BIC = \ln(\sigma_e^2) + \frac{k}{n}\ln(n)$$

  - Note compares estimated models when *x* is constant

Today

- Accuracy
- Significance Testing
- F-Measure
- AIC/BIC

Next Time

- Regression Evaluation
- Cluster Evaluation